12 Here are the first four terms of a sequence of fractions.

$$\frac{1}{1} \qquad \frac{2}{3} \qquad \frac{3}{5} \qquad \frac{4}{7}$$

The numerators of the fractions form the sequence of whole numbers   1   2   3   4 …
The denominators of the fractions form the sequence of odd numbers   1   3   5   7 …

(a) Write down an expression, in terms of n, for the nth term of this sequence of fractions.

....

(2)

(b) Using algebra, prove that when the square of any odd number is divided by 4 the remainder is 1

(3)

(Total for Question 12 is 5 marks)

15 Prove algebraically that the product of any two odd numbers is always an odd number.

(Total for Question 15 is 4 marks)

15 Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

(Total for Question 15 is 3 marks)

17 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

(Total for Question 17 is 3 marks)

17 The table gives information about the first six terms of a sequence of numbers.

Term number	1	2	3	4	5	6
Term of sequence	$\dfrac{1 \times 2}{2}$	$\dfrac{2 \times 3}{2}$	$\dfrac{3 \times 4}{2}$	$\dfrac{4 \times 5}{2}$	$\dfrac{5 \times 6}{2}$	$\dfrac{6 \times 7}{2}$

Prove algebraically that the sum of any two consecutive terms of this sequence is always a square number.

(Total for Question 17 is 4 marks)

17 Prove that the difference between two consecutive square numbers is always an odd number.
Show clear algebraic working.

(Total for Question 17 is 3 marks)

18 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2

Show clear algebraic working.

(Total for Question 18 is 3 marks)

19 Prove algebraically that, for any three consecutive even numbers,

the sum of the squares of the smallest even number and the largest even number is 8 more than twice the square of the middle even number.

(Total for Question 19 is 3 marks)

19 $a = \dfrac{14}{3x - 7}$ $\qquad$ $x = \dfrac{7}{4y - 3}$

Express a in the form $\dfrac{py + q}{ry + s}$ where p, q, r and s are integers.

Give your answer in its simplest form.

$a =$ ....

(Total for Question 19 is 3 marks)

Turn over ▶

21 (a) Show that $x(x-1)(x+1) = x^3 - x$

(1)

(b) Prove that the difference between a whole number and the cube of this number is always a multiple of 6

(3)

(Total for Question 21 is 4 marks)

21 $(2x + 23)$, $(8x + 2)$ and $(20x – 52)$ are three consecutive terms of an arithmetic sequence.

Prove that the common difference of the sequence is 12

(Total for Question 21 is 4 marks)

25 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

(Total for Question 25 is 3 marks)