16

(b) Express $x^2 - 10x + 40$ in the form $(x + a)^2 + b$, where a and b are integers.

$x^2 - 10x + 40 = (x-5)^2 - 25 + 40$

$= (x-5)^2 + 15$

(2)

(Total for Question 16 is 5 marks)

17

(c) Express $4x^2 - 8x + 7$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

$4x^2 - 8x + 7 = 4\left[x^2 - 2x + \frac{7}{4}\right]$

$= 4\left[(x-1)^2 - 1 + \frac{7}{4}\right]$

$= 4\left[(x-1)^2 + \frac{3}{4}\right]$

$= 4(x-1)^2 + 3$

(3)

(Total for Question 17 is 8 marks)

19

Given that a, b and c are integers,

(b) express $3x^2 + 12x + 19$ in the form $a(x + b)^2 + c$

$3x^2 + 12x + 19 = 3\left[x^2 + 4x + \frac{19}{3}\right]$

$= 3\left[(x+2)^2 - 4 + \frac{19}{3}\right]$

$= 3\left[(x+2)^2 + \frac{7}{3}\right]$

$= 3(x+2)^2 + 7$

(2)

(Total for Question 19 is 3 marks)

19 Express $3x^2 - 6x + 5$ in the form $a(x - b)^2 + c$

$3x^2 - 6x + 5 = 3\left[x^2 - 2x + \frac{5}{3}\right]$

$= 3\left[(x-1)^2 - 1 + \frac{5}{3}\right]$

$= 3\left[(x-1)^2 + \frac{2}{3}\right]$

$= 3(x-1)^2 + 2$

(Total for Question 19 is 3 marks)

20 (a) Write $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$

$3x^2 - 12x + 7 = 3\left[x^2 - 4x + \frac{7}{3}\right]$

$= 3\left[(x-2)^2 - 4 + \frac{7}{3}\right]$

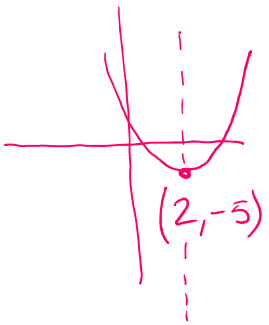
$= 3\left[(x-2)^2 - \frac{5}{3}\right]$

$= (x-2)^2 - 5$

(3)

The line **L** is the line of symmetry of the curve with equation $y = 3x^2 - 12x + 7$

(b) Using your answer to part (a) or otherwise, write down an equation of **L**.



$x = 2$

(1)

(Total for Question 20 is 4 marks)

20 (a) Express $2x^2 - 11x + 9$ in the form $a(x - b)^2 - c$ where a, b and c are numbers to be found.

$2x^2 - 11x + 9 = 2\left[x^2 - \frac{11}{2}x + \frac{9}{2}\right]$

$= 2\left[\left(x - \frac{11}{4}\right)^2 - \frac{121}{16} + \frac{9}{2}\right]$

$= 2\left[\left(x - \frac{11}{4}\right)^2 - \frac{49}{16}\right]$

$= 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$

.....

(3)

The curve **C** has equation $y = 2(x - 3)^2 - 11(x - 3) + 9$

The point P is the minimum point on **C**

(b) Find the coordinates of P

minimum point of $2x^2 - 11x + 9 \rightarrow \left(\frac{11}{4}, -\frac{49}{8}\right)$

C is translation $(x - 3)$ = 3 units right

$P \rightarrow \left(\frac{11}{4} + 3, -\frac{49}{8}\right)$

$\rightarrow \left(\frac{23}{4}, -\frac{49}{8}\right)$

(..... , ...)

(2)

(Total for Question 20 is 5 marks)

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21 The function f is such that $f(x) = 5 + 6x - x^2$ for $x \leqslant 3$

(a) Express $5 + 6x - x^2$ in the form $p - (x - q)^2$ where p and q are constants.

$5 + 6x - x^2 = -x^2 + 6x + 5$

$= -[x^2 - 6x - 5]$

$= -[(x-3)^2 - 9 - 5]$

$= -[(x-3)^2 - 14]$

$= -(x-3)^2 + 14 \quad = \quad 14 - (x-3)^2$

(2)

(b) Using your answer to part (a), find the range of values of x for which $f^{-1}(x)$ is positive.

$y = 5 + 6x - x^2$

$y = 14 - (x-3)^2$

$y + (x-3)^2 = 14$

$(x-3)^2 = 14 - y$

$x - 3 = \pm\sqrt{14-y}$

$x = 3 \pm \sqrt{14-y}$

$f^{-1}(x) = 3 \pm \sqrt{14-x}$

$x \leq 3$

$f^{-1}(x) = 3 - \sqrt{14-x}$

if $x = 5$

$3 - \sqrt{14-5} = 0$

$\sqrt{}$ can't be negative

$x > 5$ $\quad$ $x \leq 14$

$5 < x \leq 14$

(5)

(Total for Question 21 is 7 marks)

22 Write $5 + 12x - 2x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

$5 + 12x - 2x^2 = -(2x^2 - 12x - 5)$

$= -2\left[x^2 - 6x - \frac{5}{2}\right]$

$= -2\left[(x-3)^2 - 9 - \frac{5}{2}\right]$

$= -2\left[(x-3)^2 - \frac{23}{2}\right]$

$= -2(x-3)^2 + 23$

$= 23 - 2(x-3)^2$

.....

(Total for Question 22 is 4 marks)

22 The curve **S** has equation $y = f(x)$ where $f(x) = x^2$
The curve **T** has equation $y = g(x)$ where $g(x) = 2x^2 - 12x + 13$

By writing $g(x)$ in the form $a(x - b)^2 - c$, where a, b and c are constants, describe fully a series of transformations that map the curve **S** onto the curve **T**.

$g(x) = 2x^2 - 12x + 13$

$= 2\left[x^2 - 6x + \frac{13}{2}\right]$

$= 2\left[(x-3)^2 - 9 + \frac{13}{2}\right]$

$= 2\left[(x-3)^2 - \frac{5}{2}\right]$

$= 2(x-3)^2 - 5$ ← down 5 units

↑ stretch in y x2

↑ right 3 units

(Total for Question 22 is 4 marks)

23 Express $7 - 12x - 2x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

$7 - 12x - 2x^2 = -2x^2 - 12x + 7$

$= -2\left[x^2 + 6x - \frac{7}{2}\right]$

$= -2\left[(x+3)^2 - 9 - \frac{7}{2}\right]$

$= -2\left[(x+3)^2 - \frac{25}{2}\right]$

$= -2(x+3)^2 + 25$

$= 25 - 2(x+3)^2$

(Total for Question 23 is 3 marks)

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23 (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

$2x^2 - 12x + 3 = 2\left[x^2 - 6x + \frac{3}{2}\right]$

$= 2\left[(x-3)^2 - 9 + \frac{3}{2}\right]$

$= 2\left[(x-3)^2 - \frac{15}{2}\right]$

$= 2(x-3)^2 - 15$

.....

(3)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point M is the minimum point on **C**

(b) Find the coordinates of M

$2x^2 - 12x + 3$ has minimum at

$2(x-3)^2 - 15$ $\quad (3, -15)$

$y = 2(x+4)^2 - 12(x+4) + 3$

translation 4 units left

$(3, -15) \rightarrow (3-4, -15)$

$= (-1, 15)$

(..... , ...)

(2)

(Total for Question 23 is 5 marks)

24 (a) Write $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

$7 + 12x - 3x^2 = -3x^2 + 12x + 7$

$= -3\left[x^2 - 4x - \frac{7}{3}\right]$

$= -3\left[(x-2)^2 - 4 - \frac{7}{3}\right]$

$= -3\left[(x-2)^2 - \frac{19}{3}\right]$

$= -3(x-2)^2 + 19$

$= 19 - 3(x-2)^2$

.....

(4)

The curve **C** has equation $y = 7 + 12x - 3x^2$
The point A is the turning point on **C**.

(b) Using your answer to part (a), write down the coordinates of A.

$(2, 19)$

(..... , ...)

(1)

(Total for Question 24 is 5 marks)

24 Express each of a, b and c in terms of q so that

$$q + 12x - qx^2$$

can be written as $a - b(x - c)^2$

$q + 12x - qx^2 = -qx^2 + 12x + q$

$= -q\left[x^2 - \frac{12}{q}x - 1\right]$

$= -q\left[\left(x - \frac{6}{q}\right)^2 - \frac{36}{q^2} - 1\right]$

$= -q\left[\left(x - \frac{6}{q}\right)^2 - \left(\frac{36 - q^2}{q^2}\right)\right]$

$= -q\left(x - \frac{6}{q}\right)^2 + \frac{36 - q^2}{q}$

$= -q\left(x - \frac{6}{q}\right)^2 + \frac{36}{q} + q$

$a = \frac{36}{q} + q \quad b = q \quad c = \frac{6}{q}$

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 24 is 4 marks)

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24 (a) Express $7 - 4x - x^2$ in the form $p - (x + q)^2$ where p and q are constants.

$7 - 4x - x^2 = -x^2 - 4x + 7$

$= -[x^2 + 4x - 7]$

$= -[(x+2)^2 - 4 - 7]$

$= -[(x+2)^2 - 11]$

$= -(x+2)^2 + 11$

$= 11 - (x+2)^2$

.....

(2)

(b) Use your answer to part (a) to solve the equation $7 - 4(y + 3) - (y + 3)^2 = 0$

Give your solutions in the form $e \pm \sqrt{f}$ where e and f are integers.

$x = y + 3$

$11 - ((y+3) + 2)^2 = 0$

$11 = (y+5)^2$

$\pm\sqrt{11} = y + 5$

$-5 \pm \sqrt{11} = y$

.....

(3)

The curve **C** has equation $y = 3 - 5(x + 1)^2$
The point A is the maximum point on **C**.

(c) Write down the coordinates of A.

$(-1, 3)$

(..... , ...)

(1)

(Total for Question 24 is 6 marks)

25 $f(x) = 17 - 3x^2 + 12x$

Write $f(x)$ in the form $a - b(x - c)^2$ where a, b and c are constants.

$17 - 3x^2 + 12x = -3x^2 + 12x + 17$

$= -3\left[x^2 - 4x - \frac{17}{3}\right]$

$= -3\left[(x-2)^2 - 4 - \frac{17}{3}\right]$

$= -3\left[(x-2)^2 - \frac{29}{3}\right]$

$= -3(x-2)^2 + 29$

$= 29 - 3(x-2)^2$

$f(x) =$ .....

(Total for Question 25 is 4 marks)

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